[10]. Also it indicates that the gyroelectric property is similar to that of gyromagnetics.

IV. Conclusion

In this work, a gyrotropic node has been proposed and derived for the TLM technique considering general anisotropy in the frequency domain. An efficient and accurate TLM algorithm using the proposed node has been used to the study of a class of generalized planar structures involving ferrite and semiconductor layers magnetized by applying an arbitrarily oriented external dc magnetic field. The frequency-dependent characteristics of r-cut sapphire-based CPW are also obtained. It is believed that the present field-theoretical modeling technique paves the way to the unified analysis and design of microwave and millimeter-wave integrated nonreciprocal devices and high-Tc superconducting devices.

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Electromagnetic Boundary Value Problem in the Presence of a Partly Lossy Dielectric: Considerations About the Uniqueness of the Solution

S. Caorsi and M. Raffetto

Abstract—**This paper deals with the uniqueness of the solution of a boundary value problem defined by specifying the tangential components of the electric field over the closed regular boundary (or the tangential components of the magnetic field over the boundary, or the former components over part of the boundary and the latter components over the rest of the boundary) of a limited region containing a linear dielectric material not lossy everywhere. In particular, the uniqueness of the solution is proved in the case where the dielectric is everywhere linear, homogeneous, and lossless, except for a subregion where the dielectric is lossy, linear but not necessarily homogeneous.**

I. Introduction

Many authors have addressed the problem of the uniqueness of the solution of an electromagnetic boundary value problem. As many different types of problems are important in electromagnetics, a lot of different uniqueness theorems, tailored to the specific applications, have been devised. For example, Müller [1], Harrington [2], Balanis [3], and Collin [4] considered time-harmonic electromagnetic fields, and Stratton [5] dealt with arbitrarily time-varying fields.

As far as time-harmonic electromagnetic fields are concerned, it is well known [2], [3] that the tangential components of the electric field or the tangential components of the magnetic field over the boundary (or the former components over part of the boundary and the latter components over the rest of the boundary) of a domain filled with a linear and everywhere lossy dielectric uniquely determines the solution to the boundary value problem. However, to our best knowledge, nobody has proved the uniqueness of the solution when the boundary conditions are the same as described above but the dielectric is made up partly of a lossless dielectric material and partly of a lossy medium. This paper deals specifically with this problem. In particular, it will be shown that the tangential components of the electric field over the boundary (or the tangential components of the magnetic field over the boundary, or the former components over part of the boundary and the latter components over the rest of the boundary) uniquely determine the solution of the corresponding boundary value problem when the medium within the boundary is linear, homogeneous, and lossless, except for a linear and lossy subregion that may be inhomogeneous. Future efforts will be devoted to the general problem in which the linear and lossless dielectric is inhomogeneous (even with jump discontinuities).

II. Definition of the Boundary Value Problem and Uniqueness of Its Solution

Fig. 1 shows the typical boundary value problem considered in this paper. Ω denotes the region of interest, S is its regular boundary, Ω_σ is the region where the dielectric is linear, lossy but not necessarily homogeneous, and S_σ is the regular boundary of Ω_σ. In $\Omega - \Omega_\sigma$ the dielectric is assumed to be linear, homogeneous, and lossless. The

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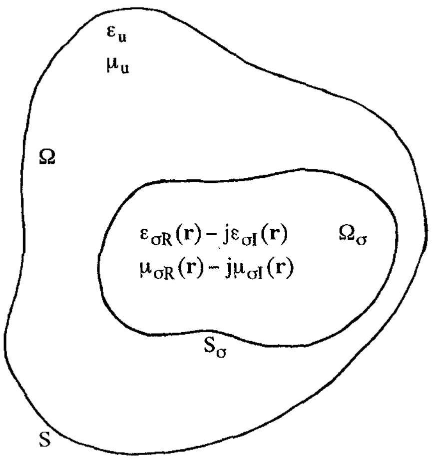

Fig. 1. The boundary value problem considered involves a linear, homogeneous, and lossless material in $\Omega - \Omega_\sigma$ and a linear, lossy but possibly inhomogeneous dielectric in Ω_σ.

known boundary conditions specify the tangential electric components over S (or the tangential magnetic components over S, or the former components over part of S and the latter components over the rest of S).

Thus, the mathematical model we consider is

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\dot{\mu}(\boldsymbol{r})\boldsymbol{H} & \text{in } \Omega \\ \nabla \times \boldsymbol{H} = j\omega\dot{\varepsilon}(\boldsymbol{r})\boldsymbol{E} & \text{in } \Omega \qquad \omega > 0 \\ \boldsymbol{n} \times \boldsymbol{E} = \boldsymbol{G} & \text{on } S \end{cases} \tag{1}$$

where $\dot{\varepsilon}(\boldsymbol{r})$ is the complex dielectric permittivity

$$\dot{\varepsilon}(\boldsymbol{r}) = \begin{cases} \varepsilon_u & \text{in } \Omega - \Omega_\sigma \\ \varepsilon_{\sigma R}(\boldsymbol{r}) - j\varepsilon_{\sigma I}(\boldsymbol{r}) & \text{in } \Omega_\sigma \end{cases} \tag{2}$$

$\dot{\mu}(\boldsymbol{r})$ is the complex magnetic permeability

$$\dot{\mu}(\boldsymbol{r}) = \begin{cases} \mu_u & \text{in } \Omega - \Omega_\sigma \\ \mu_{\sigma R}(\boldsymbol{r}) - j\mu_{\sigma I}(\boldsymbol{r}) & \text{in } \Omega_\sigma \end{cases} \tag{3}$$

ω is the angular frequency and $\boldsymbol{n}$ is the unit vector normal to S and pointing outward from region Ω.

It is important to recall that the solution of the linear boundary value problem (1) is unique if and only if the corresponding homogeneous problem

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\dot{\mu}(\boldsymbol{r})\boldsymbol{H} & \text{in } \Omega \\ \nabla \times \boldsymbol{H} = j\omega\dot{\varepsilon}(\boldsymbol{r})\boldsymbol{E} & \text{in } \Omega \qquad \omega > 0 \\ \boldsymbol{n} \times \boldsymbol{E} = 0 & \text{on } S \end{cases} \tag{4}$$

admits only the trivial solution.

In order to prove that (4), in the case considered, admits only the trivial solution $\boldsymbol{E} = \boldsymbol{H} = 0$ in Ω, let us apply the typical procedure [2], [3] used to prove the uniqueness of the solution, i.e., the energy conservation law or Poynting's theorem.

The continuity of the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ within the boundary S implies that $\boldsymbol{E} \times \boldsymbol{H}^*$ (* indicates complex conjugate) has a continuous normal component over the possible internal interfaces. Consequently, the divergence theorem can be applied to the whole domain Ω

$$\oint_S \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{n}\, dS = \int_\Omega \nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}^*)\, dV. \tag{5}$$

By using the vector identity

$$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B}) \tag{6}$$

we obtain

$$\oint_S \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{n}\, dS = \int_\Omega \boldsymbol{H}^* \cdot (\nabla \times \boldsymbol{E}) - \boldsymbol{E} \cdot (\nabla \times \boldsymbol{H}^*)\, dV \tag{7}$$

and, by using Maxwell's equations

$$\begin{aligned} &\oint_S \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{n}\, dS \\ &\quad = \int_\Omega \boldsymbol{H}^* \cdot (-j\omega\dot{\mu}(\boldsymbol{r})\boldsymbol{H}) - \boldsymbol{E} \cdot (-j\omega\dot{\varepsilon}^*(\boldsymbol{r})\boldsymbol{E}^*)\, dV \\ &\quad = \int_\Omega -j\omega\dot{\mu}(\boldsymbol{r})|\boldsymbol{H}|^2 + j\omega\dot{\varepsilon}^*(\boldsymbol{r})|\boldsymbol{E}|^2\, dV. \end{aligned} \tag{8}$$

Since $\boldsymbol{n} \times \boldsymbol{E} = 0$ over S (or $\boldsymbol{n} \times \boldsymbol{H} = 0$ over S, or $\boldsymbol{n} \times \boldsymbol{E} = 0$ over part of S and $\boldsymbol{n} \times \boldsymbol{H} = 0$ over the rest of S), we have

$$\oint_S \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{n}\, dS = 0 \tag{9}$$

and consequently

$$0 = \int_\Omega -j\omega\dot{\mu}(\boldsymbol{r})|\boldsymbol{H}|^2 + j\omega\dot{\varepsilon}^*(\boldsymbol{r})|\boldsymbol{E}|^2\, dV. \tag{10}$$

Substituting for $\dot{\varepsilon}(\boldsymbol{r})$ from (2) and for $\dot{\mu}(\boldsymbol{r})$ from (3) we obtain

$$\begin{aligned} 0 &= \int_{\Omega-\Omega_\sigma} -j\omega\mu_u|\boldsymbol{H}|^2 + j\omega\varepsilon_u|\boldsymbol{E}|^2\, dV + \int_{\Omega_\sigma} -j\omega(\mu_{\sigma r}(\boldsymbol{r}) \\ &\quad - j\mu_{\sigma I}(\boldsymbol{r}))|\boldsymbol{H}|^2 + j\omega(\varepsilon_{\sigma R}(\boldsymbol{r}) + j\varepsilon_{\sigma I}(\boldsymbol{r}))|\boldsymbol{E}|^2\, dV \\ &= \int_{\Omega-\Omega_\sigma} -j\omega\mu_u|\boldsymbol{H}|^2 + j\omega\varepsilon_u|\boldsymbol{E}|^2\, dV + \int_{\Omega_\sigma} -j\omega\mu_{\sigma R}(\boldsymbol{r}) \\ &\quad \cdot |\boldsymbol{H}|^2 + j\omega\varepsilon_{\sigma R}(\boldsymbol{r})|\boldsymbol{E}|^2\, dV - \int_{\Omega_\sigma} \omega\mu_{\sigma I}(\boldsymbol{r})|\boldsymbol{H}|^2 \\ &\quad + \omega\varepsilon_{\sigma I}(\boldsymbol{r})|\boldsymbol{E}|^2\, dV. \end{aligned} \tag{11}$$

This equation implies that both the real and imaginary parts of the right-hand side integral are zeroes. Thus, in particular, the real part must be zero, i.e.,

$$\int_{\Omega_\sigma} \omega\mu_{\sigma I}(\boldsymbol{r})|\boldsymbol{H}|^2 + \omega\varepsilon_{\sigma I}(\boldsymbol{r})|\boldsymbol{E}|^2\, dV = 0. \tag{12}$$

Both terms of the integrand of (12) are greater than or equal to zero $\forall \boldsymbol{r} \in \Omega_\sigma$. As a consequence, (12) is satisfied if and only if

$$\begin{aligned} \omega\varepsilon_{\sigma I}(\boldsymbol{r})|\boldsymbol{E}|^2 &= 0 \quad \forall \boldsymbol{r} \in \Omega_\sigma \\ \omega\mu_{\sigma I}(\boldsymbol{r})|\boldsymbol{H}|^2 &= 0 \quad \forall \boldsymbol{r} \in \Omega_\sigma. \end{aligned} \tag{13}$$

Thus, in order to satisfy (12), we must have $\boldsymbol{E} = 0$ where $\omega\varepsilon_{\sigma I}(\boldsymbol{r})$ is strictly positive, and $\boldsymbol{H} = 0$ where $\omega\mu_{\sigma I}(\boldsymbol{r})$ is strictly positive.

But, by using Maxwell's equations

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -j\omega\dot{\mu}(\boldsymbol{r})\boldsymbol{H} \\ \nabla \times \boldsymbol{H} &= j\omega\dot{\varepsilon}(\boldsymbol{r})\boldsymbol{E} \end{aligned} \tag{14}$$

and

$$\begin{aligned} -j\omega\dot{\mu}(\boldsymbol{r}) &\neq 0 \quad \forall \boldsymbol{r} \in \Omega \\ j\omega\dot{\varepsilon}(\boldsymbol{r}) &\neq 0 \quad \forall \boldsymbol{r} \in \Omega \end{aligned} \tag{15}$$

we have that $\boldsymbol{E} = 0$ ($\boldsymbol{H} = 0$) in any subregion of Ω implies $\boldsymbol{H} = 0\,(\boldsymbol{E} = 0)$ in the same subregion. Consequently, $\boldsymbol{E} = \boldsymbol{H} = 0$ where $\omega\varepsilon_{\sigma I}(\boldsymbol{r})$ or $\omega\mu_{\sigma I}(\boldsymbol{r})$ are strictly positive. Since in a lossy dielectric $\varepsilon_{\sigma I}(\boldsymbol{r})$ and $\mu_{\sigma I}(\boldsymbol{r})$ cannot be both zero quantities we obtain

$$\boldsymbol{E} = \boldsymbol{H} = 0 \text{ in } \Omega_\sigma. \tag{16}$$

However, (11) does not provide any information about $\boldsymbol{E}$ or $\boldsymbol{H}$ in $\Omega - \Omega_\sigma$, where the dielectric is lossless. Consequently, the following question can be raised: is it possible to have a nonzero field in $\Omega - \Omega$. The answer is no, as proved by Müller [1] (Theorem 34).

In fact, in $\Omega - \Omega_\sigma$ the dielectric is linear and homogeneous, and, as no jump discontinuity in electrical properties is possible, $\boldsymbol{E}, \boldsymbol{H}, \nabla \times \boldsymbol{E}$, and $\nabla \times \boldsymbol{H}$ are continuous. Moreover, $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy

$$\begin{aligned}\nabla \times \boldsymbol{E} &= -j\omega\mu_u \boldsymbol{H}\\ \nabla \times \boldsymbol{H} &= j\omega\varepsilon_u \boldsymbol{E}\end{aligned} \tag{17}$$

and, by using (16) and the tangential continuity of $\boldsymbol{E}$ and $\boldsymbol{H}$ across dielectric interfaces

$$\left.\begin{aligned}\boldsymbol{n} \times \boldsymbol{E} = 0\\ \boldsymbol{n} \times \boldsymbol{H} = 0\end{aligned}\right\} \text{ on } S_\sigma. \tag{18}$$

Then ([1], theorem 34) $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish identically in $\Omega - \Omega_\sigma$, i.e.,

$$\boldsymbol{E} = \boldsymbol{H} = 0 \text{ in } \Omega - \Omega_\sigma. \tag{19}$$

Finally, (16) and (19) imply

$$\boldsymbol{E} = \boldsymbol{H} = 0 \text{ in } \Omega \tag{20}$$

and the uniqueness of the solution is proved for the present particular case.

Note that Ω_σ cannot collapse to a point, line or surface; it must be a three–dimensional (3-D) domain bounded by a regular surface.

III. Conclusion

A generalization of the standard uniqueness theorem for time-harmonic electromagnetic fields has been presented and proved. In particular, it has been shown that a linear, lossless, and homogeneous dielectric can be part of the domain of interest. It can be useful to know that, even in this case, the boundary value problem defined by specifying the tangential components of the electric field over the boundary (or the tangential components of the magnetic field over the boundary, or the former components over part of the boundary and the latter components over the rest of the boundary) has a unique solution. However, it will be important to complete this generalization by assuming the linear and lossless dielectric (which is only part of the domain) to be inhomogeneous and even to present jump discontinuities.

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Measurement of Simple Resonant Equivalent Circuits for Microstrip Antennas

Steven J. Weiss and Walter K. Kahn

***Abstract*—This paper presents a procedure which can be used to model the input admittance of a probe-fed microstrip antenna using simple circuit components. The values of the components are extracted from experimental data and represent the antenna about any resonant mode. A good circuit description of the antenna can greatly facilitate system analysis.**

I. Introduction

The *cavity model* has been of great value over the years lending practical insight into the operation of microstrip antennas. Using this model, the electromagnetic field between the patch and ground plane of the antenna (the internal field) is assumed to closely resemble the field which would be maintained by a cavity resonator having magnetic walls on the perimeter and the same electric walls as the antenna on the top and bottom [1]–[2]. The resonant modes are dependent on the geometry of the patch. This cavity-like behavior of the internal field(s) suggests that the antennas may be amenable to proven techniques, developed over the years, which are used to characterize the input admittance of cavity resonators.

This paper will develop a procedure by which measured input admittance data may be transformed to a circle of constant conductance. After this transformation is performed, it is a simple matter to realize a resonant circuit which describes the *transformed* data points. Since the transformation itself can be accomplished using circuit elements, a complete circuit description of the antenna's input admittance is obtained.

II. Transformation of the Data

Fig. 1 presents measured admittance data obtained from a probe-fed microstrip antenna using a Hewlett Packard 8720A network analyzer. The circular shape of the data is characteristic of these antennas and not dependent on the geometry of the patch [2]. The center of the circle makes an angle with the horizontal axis of the Smith chart designated by "2θ."

This analysis requires a transformation of the data to a circle of constant conductance. Such a transformation is physically realized using a length of transmission line (for rotation) and an attenuator. That is, the data may be rotated to a position symmetric about the horizontal axis of the Smith chart from the position shown in Fig. 1 if the angle θ is known. This data, symmetric about the horizontal axis, can then be viewed as originating from a circle of constant conductance attenuated by "2α". Accordingly, the transformation of the reflection coefficient data from a circle of constant conductance to a position such as that shown in Fig. 1 is realized from the relation:

$$\Gamma(\omega)_{\text{data}} = e^{-j2\theta} e^{-2\alpha} \Gamma(\omega)_{\text{constant conductance circle}} \tag{1}$$

The values of θ and α are determined from a knowledge of the center and radius of the measured data circle.

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